Scaling and spatial correlations in the quasibrittle process zone

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Cornell University

Quasibrittle materials & fracture

Brittle with quenched disorder

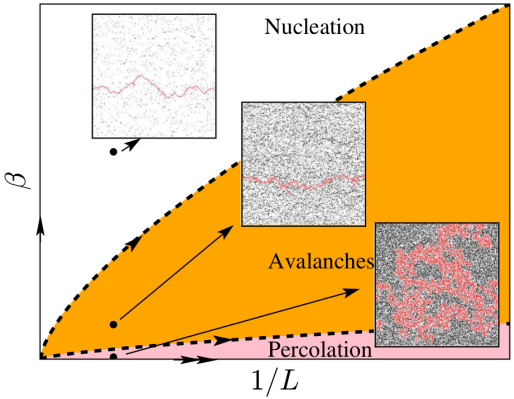
Process zone of correlated microfracture, large as meters

Size and boundary effects dominate statistics of fracture

Depending on substance and scale, fracture can look clean or crumbly

Crossover theory

Shekhawat *et al*, Phys Rev Lett **110** 185505



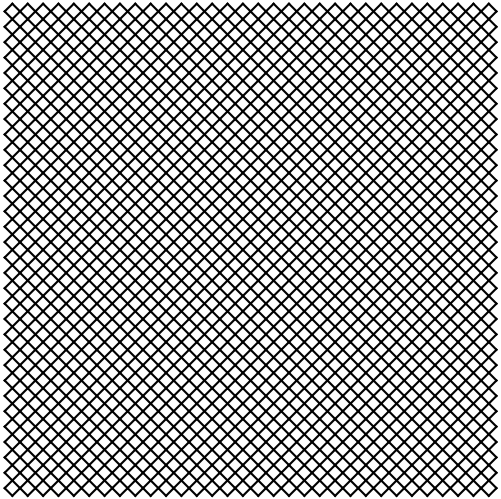
Previous work suggests a *scaling crossover* between fracture regimes

Crumbly regime controlled by percolation fixed point, clean by nucleation

Avalanches dominate intermediate disorder, vanish when first fractures system

Analogous idea for crack structure: crumbly crack surface coarse-grains to clean one through microcracked crossover

Our simulations: Fuse networks

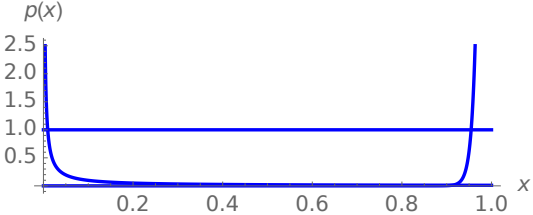


Resistive fuses as scalar elastic analogue

Quenched disorder via breaking thresholds $0 < x \leq 1$ distributed by $p(x) = \beta x^{\beta-1}$

Fractured adiabatically: fuse with largest threshold to current ratio broken

Small β is disordered, large β ordered



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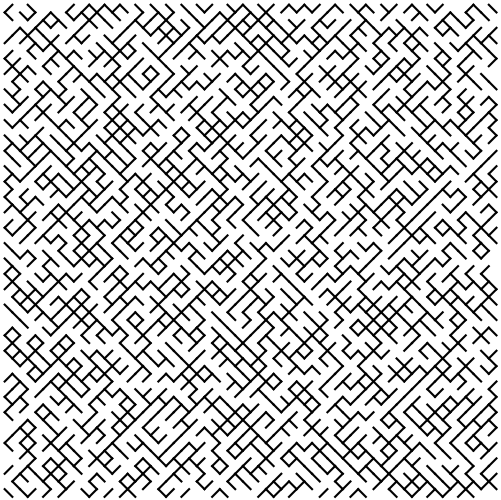

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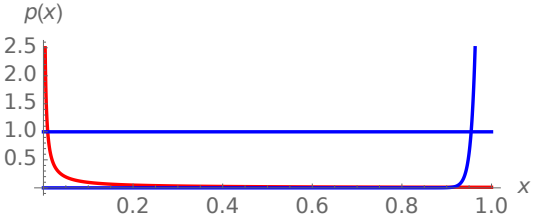

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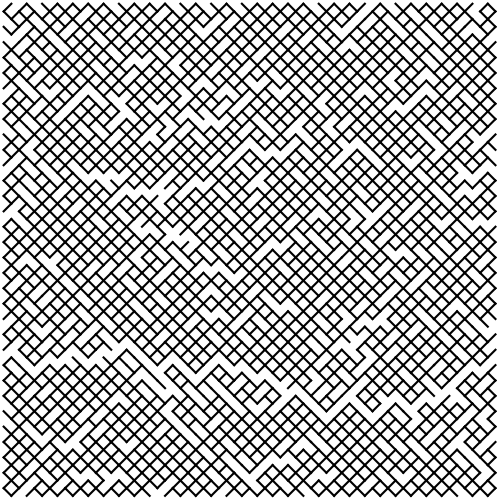

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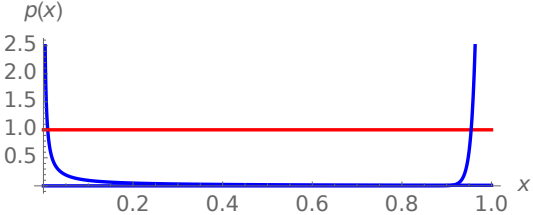

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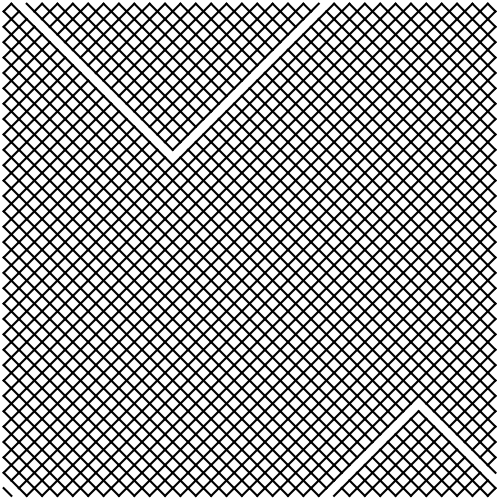

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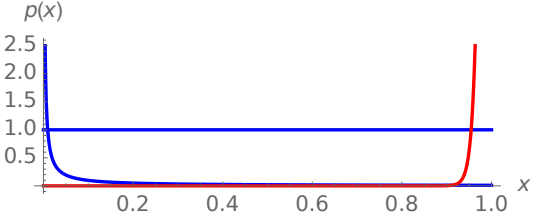

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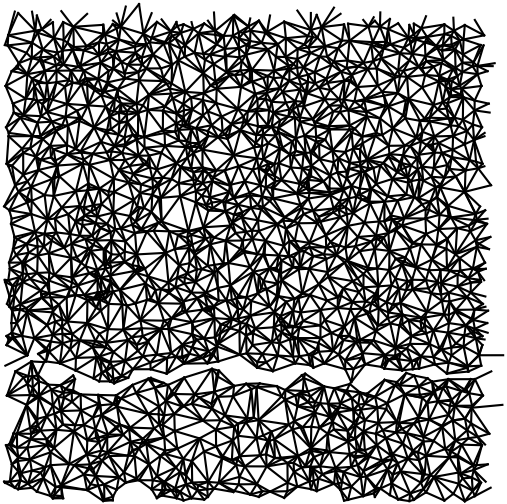

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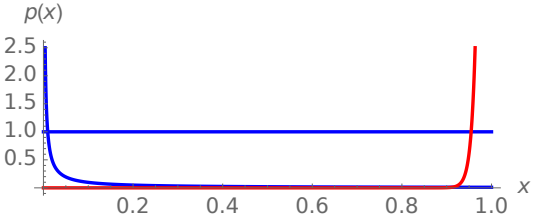
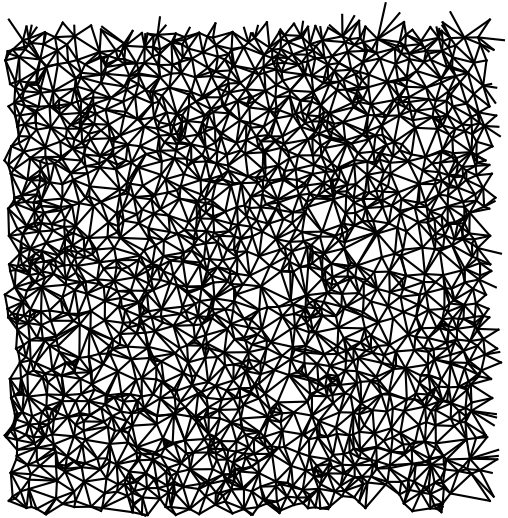


Properties of interest

Many spatial properties to study:

- backbone
- spanning cluster
- all clusters
- non-spanning clusters
- final avalanche

Focus on $g(\Delta x, \Delta y)$: probability that site displaced by $(\Delta x, \Delta y)$ is in same cluster



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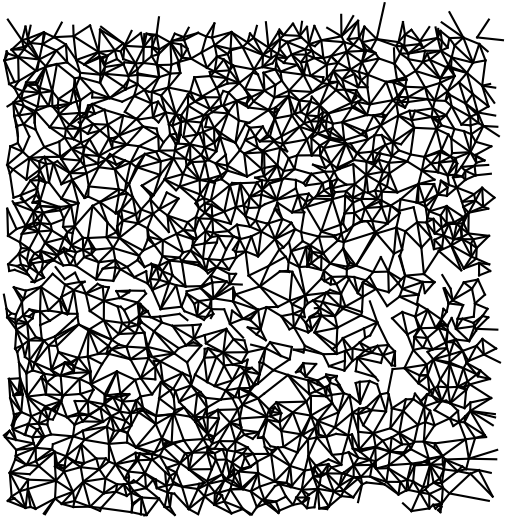

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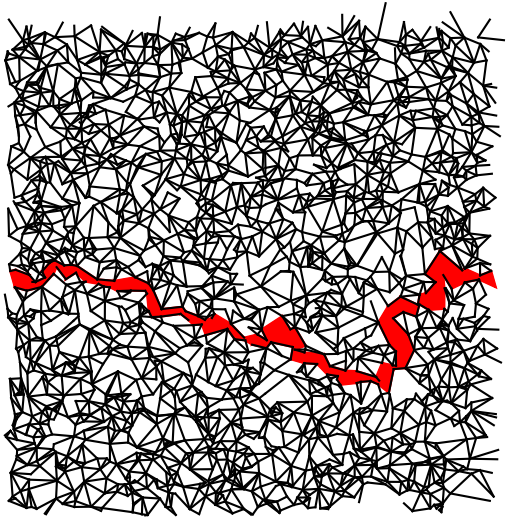

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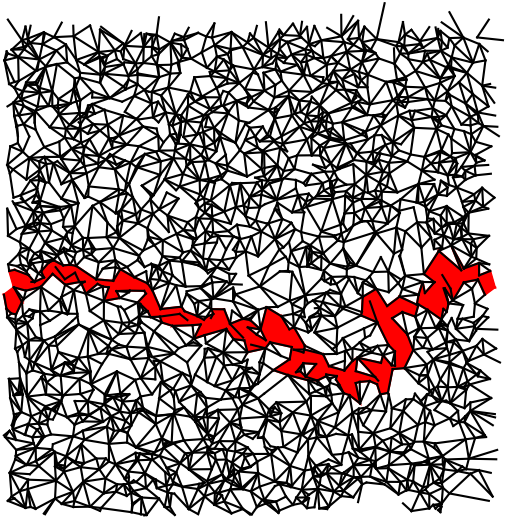

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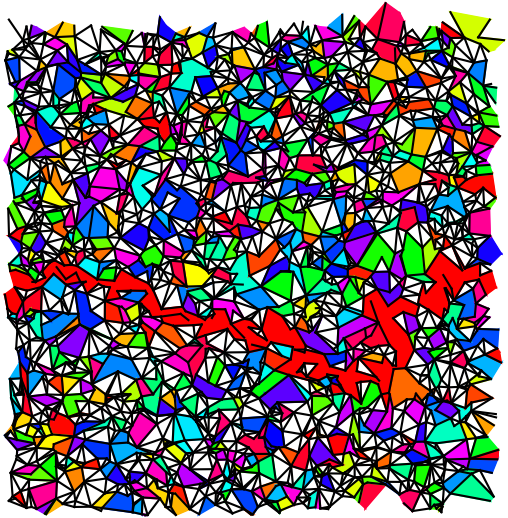

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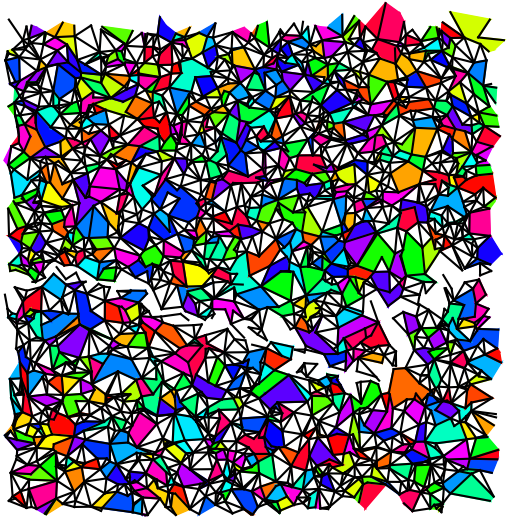

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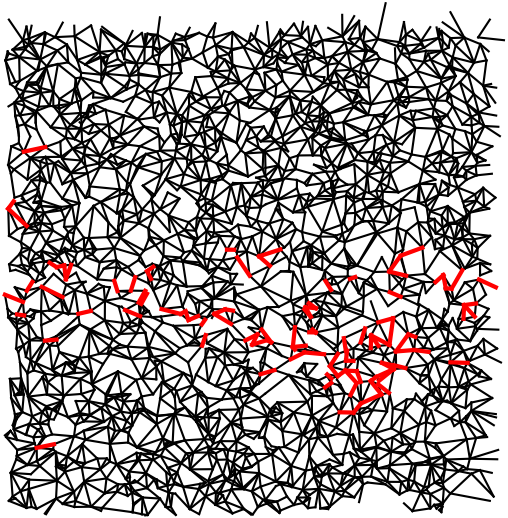

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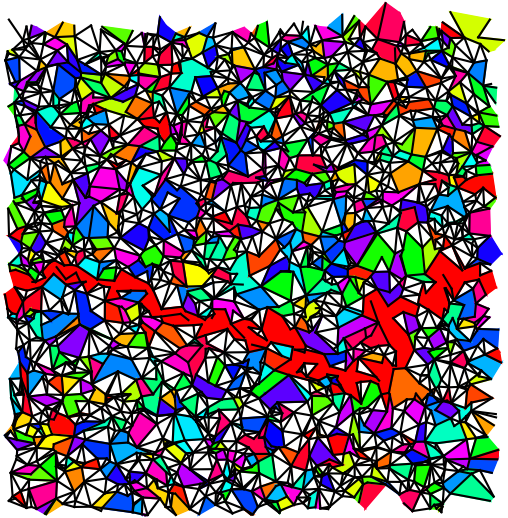

Fixed points

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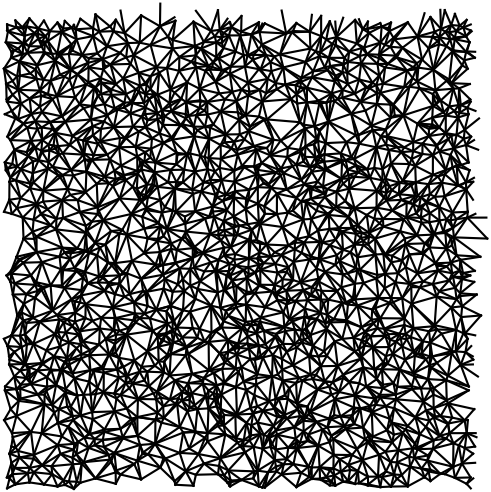

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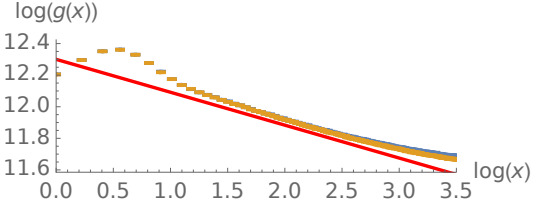

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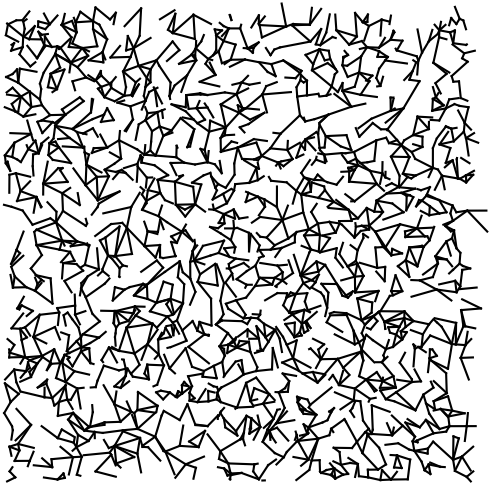

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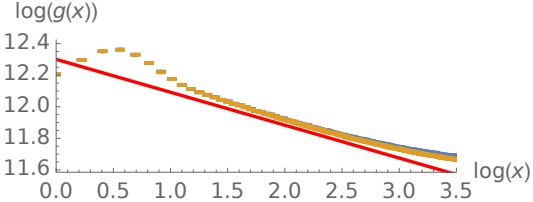

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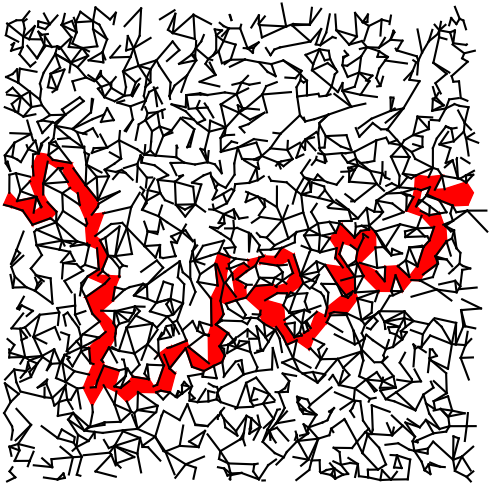

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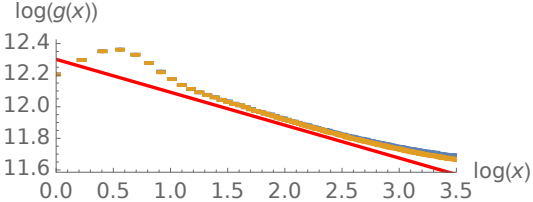

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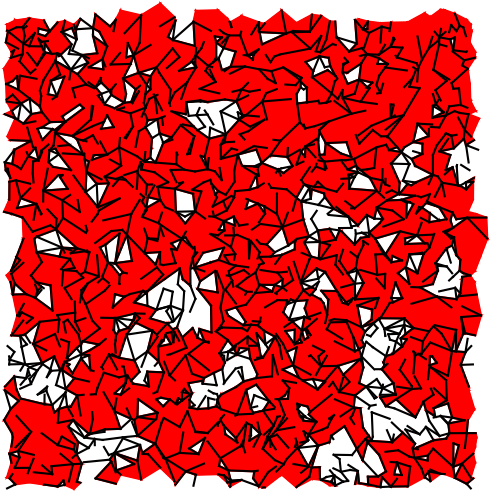

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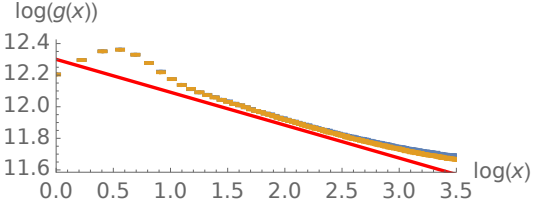

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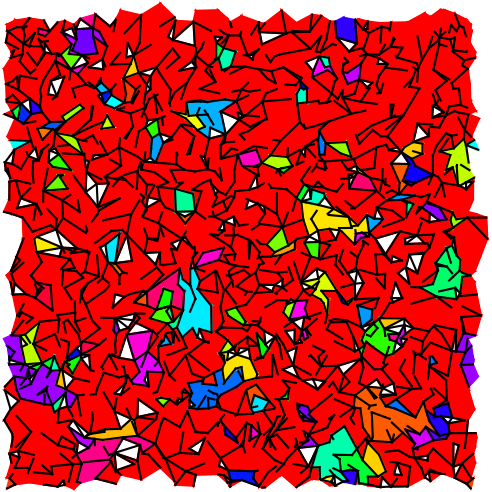

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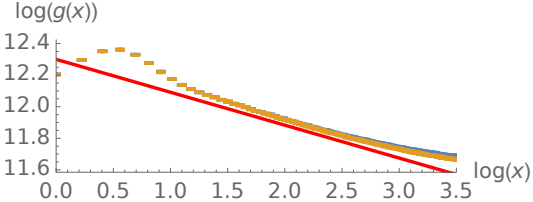

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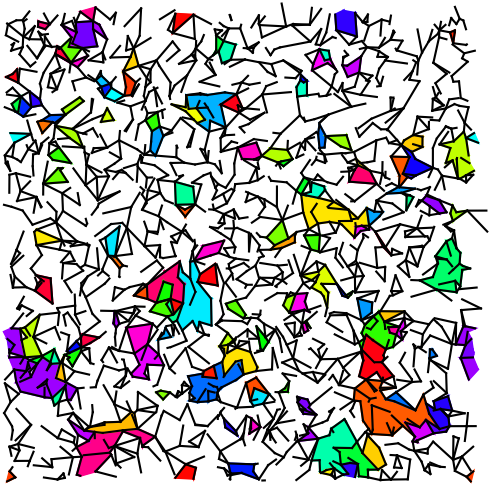

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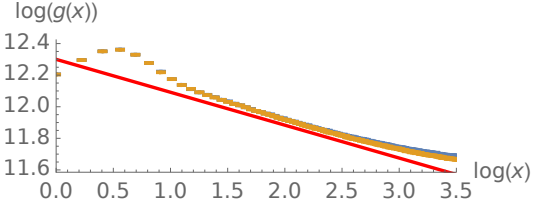

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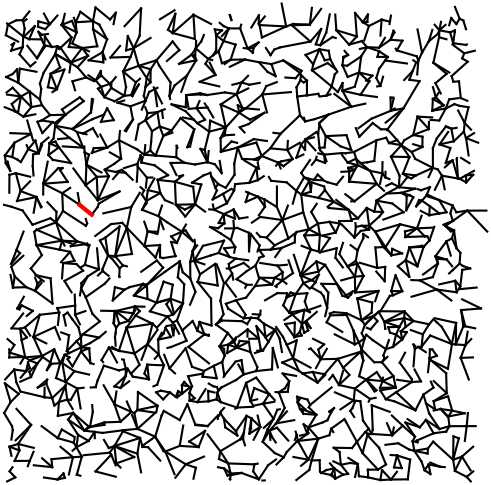

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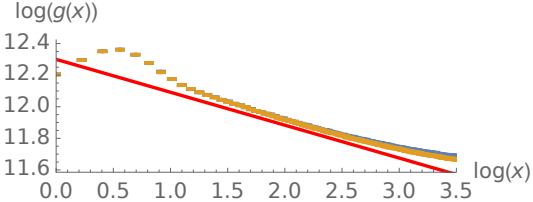

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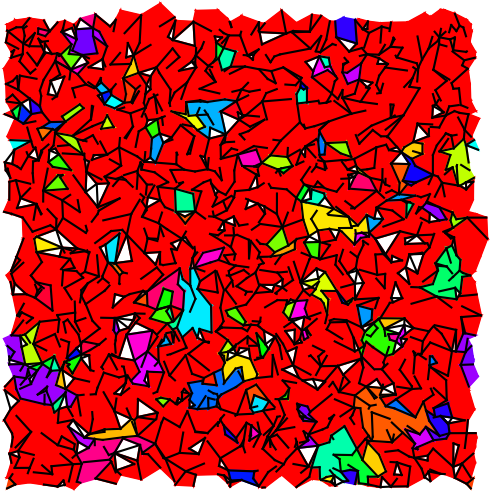

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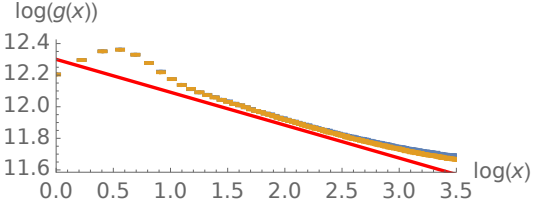

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$\beta = \infty$ – anisotropic, self-affine nucleation

As $L \to \infty$, $\beta \to \infty$, reduces to nucleated crack propagation

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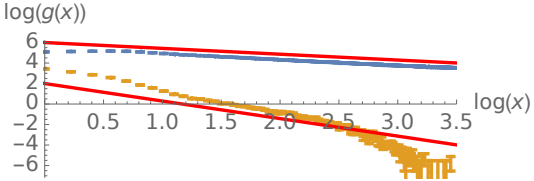

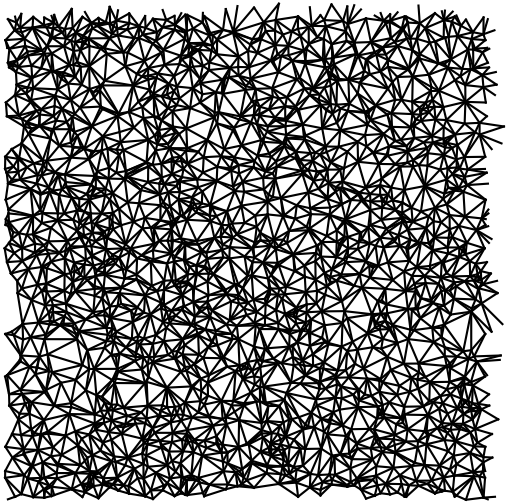

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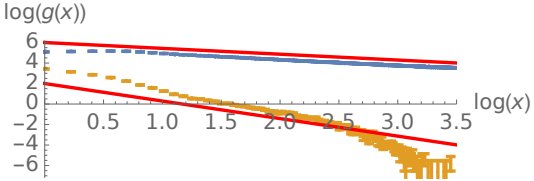

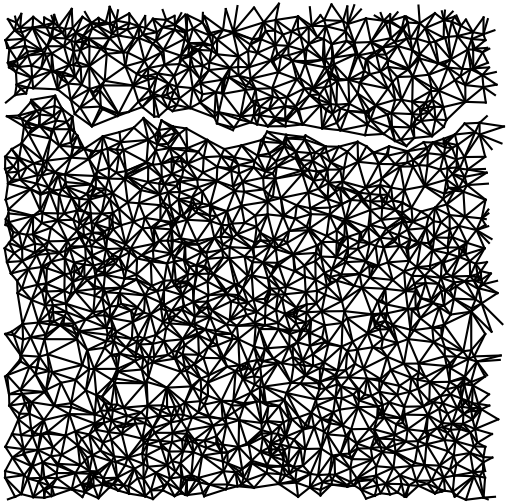

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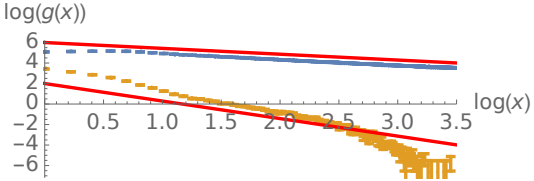

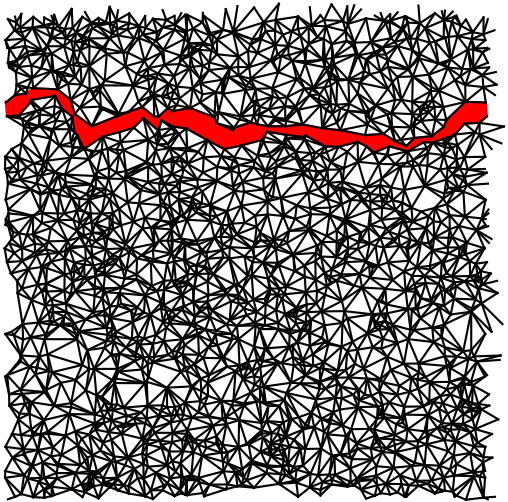

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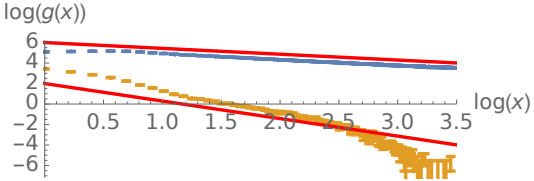

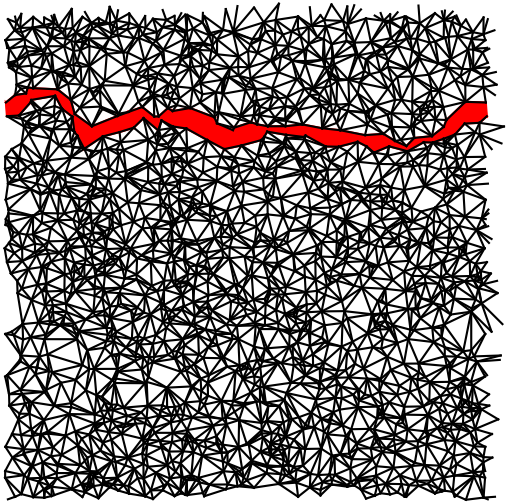

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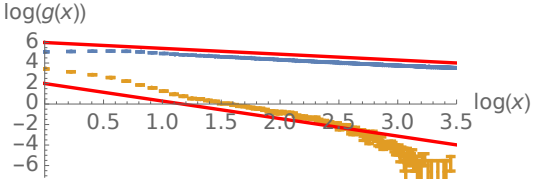

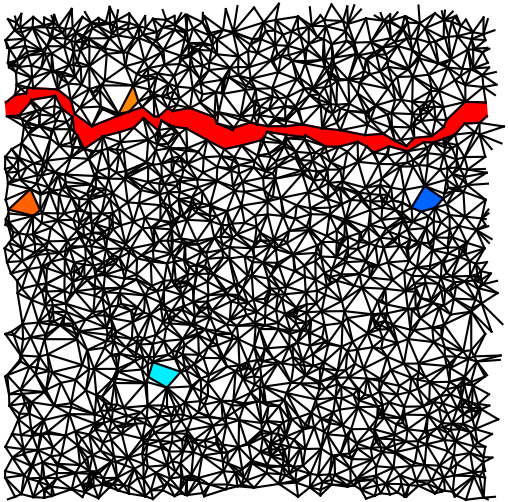

Fixed points

$\beta = \infty$ – anisotropic, self-affine nucleation

As $L \to \infty$, $\beta \to \infty$, reduces to nucleated crack propagation

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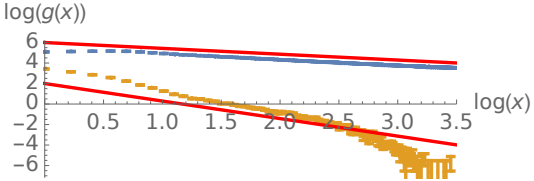

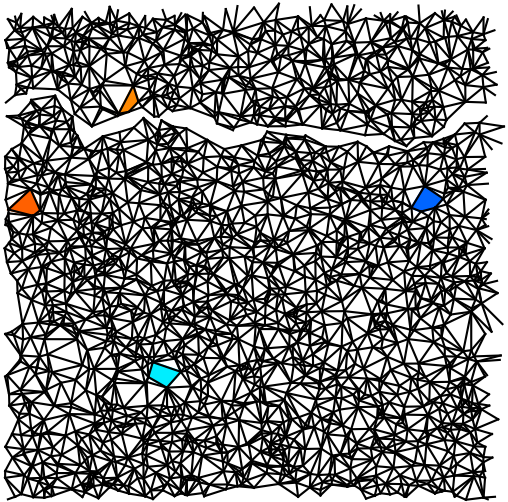

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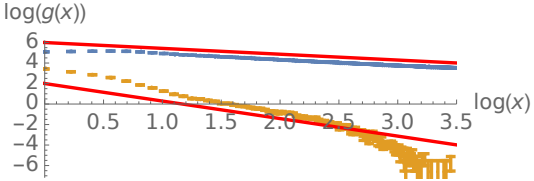

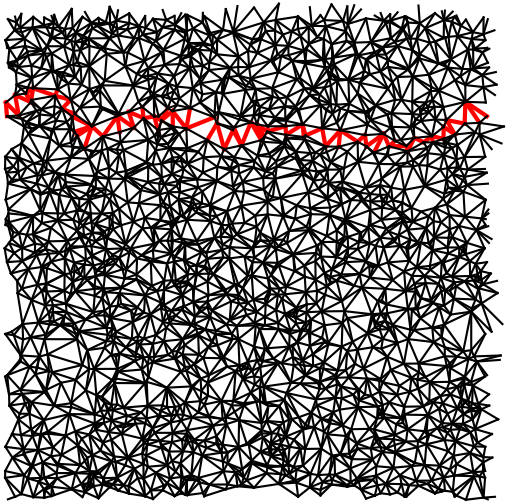

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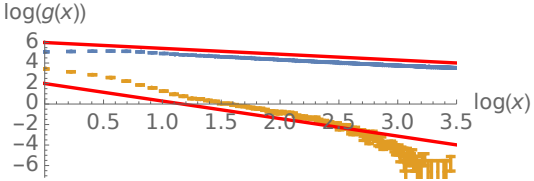
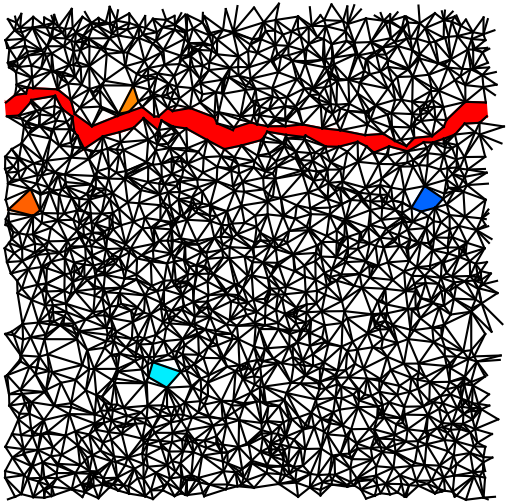
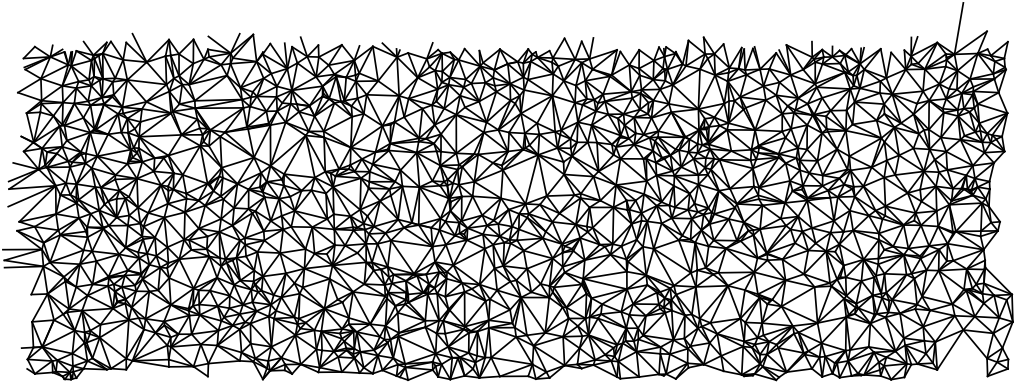




Self-similarity to self-affinity

Voltage (strain) applied along one direction—we should expect anisotropy!

Self-affine anisotropy emerges at different scales in different properties, but well within the intermediate microfractured regime.



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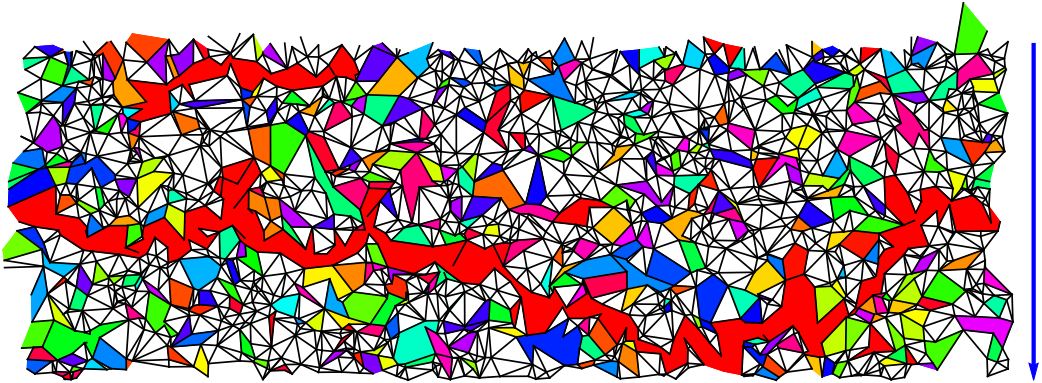

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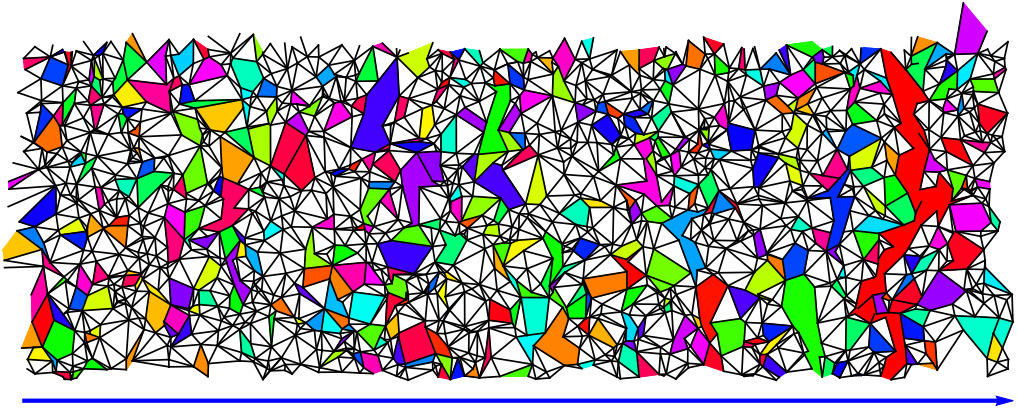
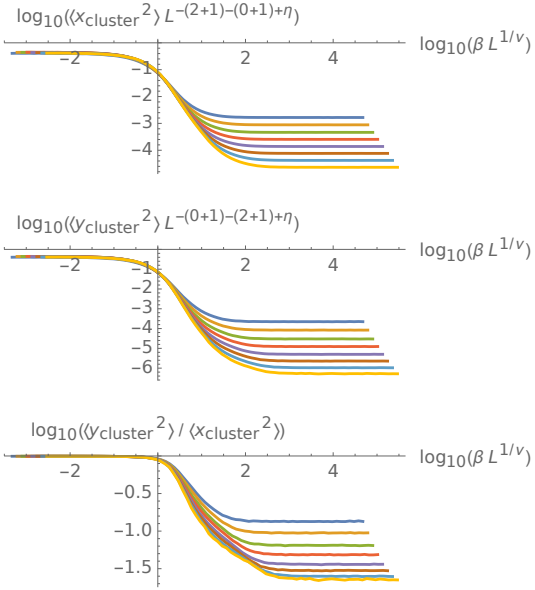


Scaling theory

Quantities like moments of g depend on $L_x \beta^{\nu_x}$ and $L_y \beta^{\nu_y}$, no simple "collapse"

Show expected isotropic percolation scaling in disordered limit, unusual crossover in the intermediate regieme.

Spatial properties of avalanches remain anisotropic in all regiemes.



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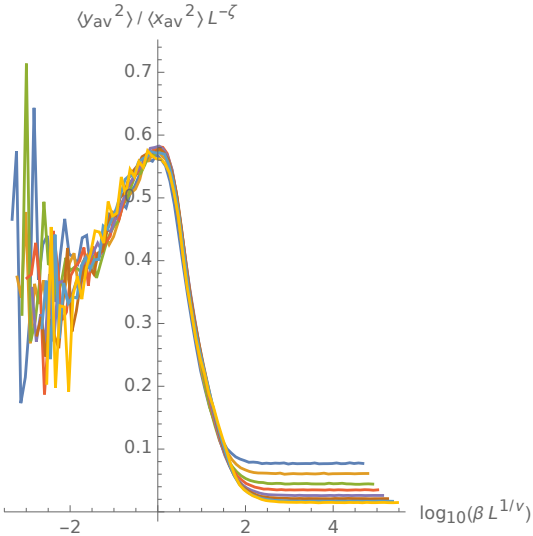
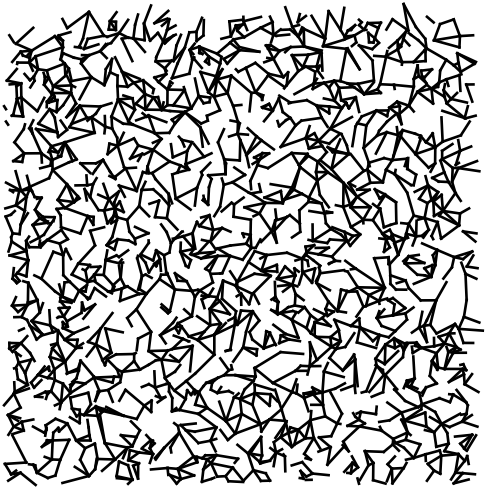


Continuing work

Need to make consistent cross-property measurement of exponents; working on expected form of scaling functions through different regiemes

How does the established multifractal distribution of bond currents (stresses) affect this analysis, if at all?



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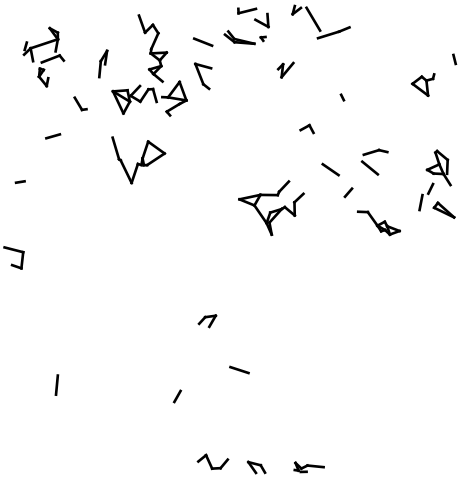

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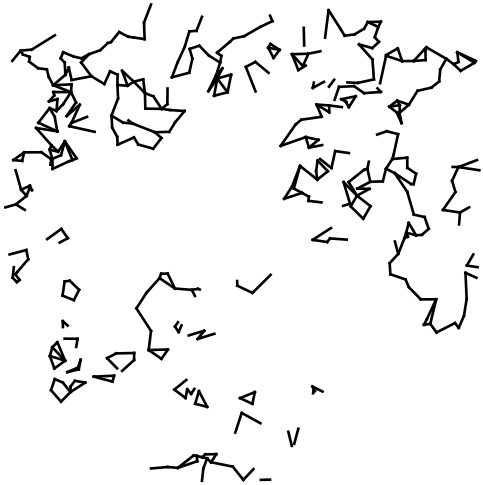

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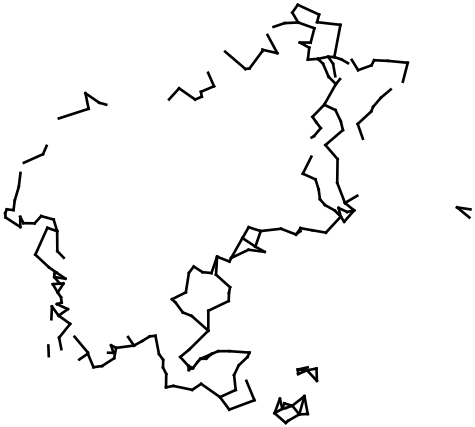

Questions?